

De Proportione Sonorum

Of the Proportion of Sounds · the divided string, and why some numbers please the ear

I

1. MONOCHORDUM

The single string, and its division into consonances

Stretch one gut over a board of 1200 millimetres between two bridges and load it with a known weight. Stop it at the half and it sounds the same voice higher — the diapason, which is 2 to 1. Stop it at two thirds and it gives the diapente, 3 to 2. At three quarters, the diatessaron, 4 to 3. These are not opinions of musicians but properties of the number, and the ear, which cannot count, nevertheless knows them.

Note that all the pleasing stops fall where the string is divided by small whole numbers, and that the pleasure fails as the numbers grow. *Coniectura: the ear is a counter of coincidences. Where the returns of two strings agree often, we call it sweetness; where they agree seldom, harshness. The judgment is arithmetic performed in the dark.*

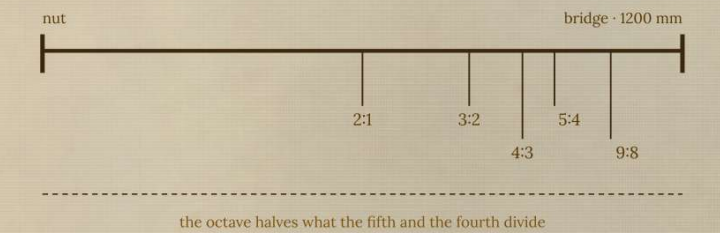


Fig. 1. The canon divided. Distances are measured from the bridge at the right hand, so that a shorter portion stands to the left of each mark.

2. COMMA PYTHAGORAE

The crack in the wheel of fifths

Mount twelve fifths one upon another and you should arrive at the seventh octave above. You do not. The twelve fifths overshoot by a small quantity which no tuning can hide and every tuning must place somewhere.

$$\left(\frac{3}{2}\right)^{12} \div 2^7 = 531441 / 524288$$

= 1.013643 · a surplus near one fifty-fifth of the octave

Coniectura: the tuner is therefore a kind of engineer, distributing an error he cannot destroy — as the vault distributes a load. Give the whole of it to one fifth and that fifth howls; spread it in twelve equal shares and all fifths lie a little, none audibly.

3. DE SONO COMPOSITO

That one string sounds many voices at once

Touch the string lightly at its middle while it sounds and the low voice vanishes, leaving the octave; touch at a third and the twelfth remains. Therefore the string was singing all of these together and the ear had chosen one to call the note. Lay small paper riders along the string and pluck: the riders at the still places stand, the others are thrown off, and so the eye reads what the ear guessed.

PARS	NODI	RATIO	VOX
1	none	1:1	the note
2	the middle	2:1	octave
3	two thirds	3:1	twelfth
4	quarters	4:1	double octave
5	fifths	5:1	+ major third
6	sixths	6:1	+ fifth
7	sevenths	7:1	a stranger

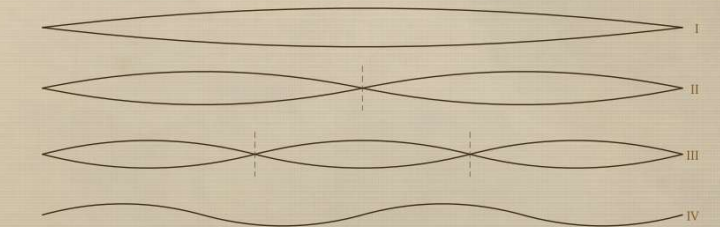


Fig. 2. The first four modes of one gut, with the still points marked. *Coniectura: every sounding body is a company of singers who never disagree about tempo.*

One further thing the riders show: the higher modes are not equally strong, and the pluck decides which. Pluck at the middle and the even modes are silent, for the middle is their still point; pluck near the bridge and all of them shout together, and the tone turns nasal and brilliant. *Coniectura: the player chooses not only the note but the company that sings it, and he does so with the position of one finger. Herein lies the whole difference between the harp and the harpsichord, which are otherwise the same machine.*

4. TABULA CONSONANTIARUM

Table of the intervals upon a canon of 1200 mm

INTERVALLUM	RATIO	LONGITUDO	PARTES
Unisonus <i>unison</i>	1:1	1200.0	0
Semitonium <i>limma</i>	256:243	1139.1	90
Tonus <i>whole tone</i>	9:8	1066.7	204
Semititonus <i>minor third</i>	6:5	1000.0	316
Ditonus <i>major third</i>	5:4	960.0	386
Diatessaron <i>fourth</i>	4:3	900.0	498
Tritonus <i>diabolus</i>	45:32	853.3	590
Diapente <i>fifth</i>	3:2	800.0	702
Sexta minor <i>minor sixth</i>	8:5	750.0	814
Sexta maior <i>major sixth</i>	5:3	720.0	884
Septima minor <i>seventh</i>	16:9	675.0	996
Septima maior <i>major seventh</i>	15:8	640.0	1088
Diapason <i>octave</i>	2:1	600.0	1200

Partes are thousandth parts of the octave, by which two intervals of unlike ratio may be added as lengths are added. Coniectura of the copyist: the master divides the octave logarithmically two generations before any man writes the logarithm.

5. DE NUMERO SORDIDO

Of the impure number — the twelfth root

If the surplus must be shared equally, then each semitone is that quantity which, taken twelve times upon itself, gives the double. It is a number no ruler can lay down and no ratio can name.

$$s = \sqrt[12]{2} = 1.059463$$

the equal semitone; the fifth becomes 700 parts, short of 702 by two

Coniectura: geometry gives what arithmetic refuses. Construct two mean proportionals and you may cut the fret; the lute player does with a compass what the counter cannot do with a pen. I hold that the irrational is not a defect of number but the place where number touches the continuous, as the tangent touches the circle at no length at all.

6. TABULA TEMPERAMENTORUM

Four tunings compared, in thousandth parts of the octave

INTERVALLUM	PYTH.	IUSTUM	MESO.	AEQU.
Semitonium	90	112	117	100
Tonus	204	204	193	200
Ditonus	408	386	386	400
Diatessaron	498	498	503	500
Diapente	702	702	697	700
Sexta maior	906	884	890	900

The Pythagorean keeps its fifths pure and pays for it with a third too wide by twenty-two parts — a whole comma, which howls in a held chord. The mesotonic gives back a quarter of a comma to every fifth and buys eight sweet thirds at the cost of one unusable interval, which the players call the wolf. The equal spreads the debt so thin that no interval is pure and none is intolerable.

Coniectura: there is no honest tuning, only a choice of which lie to tell and to whom. The organ, which cannot retune between pieces, must tell the same lie all evening; the singer, who retunes at every chord, need tell none — and this, not the sweetness of the throat, is why the unaccompanied voice is the sweetest instrument.

7. DE NUMERIS AMICIS

Of the friendly numbers, and the limit of the senario

Fra Luca holds that consonance ends at six, the senario, because the first six numbers contain every sweet interval and no other. I have tried the sevenths upon the canon and find them not harsh but strange — a third kind of sound, neither sweet nor sour, which the ear cannot place and therefore will not use.

NUM.	RATIO NOVA	PARTES	IUDICIUM
5	5:4 ditonus	386	soft, sweet
6	6:5 semiditonus	316	grave, sweet
7	7:4 septima	969	strange
7	7:5 tritonus	583	restless
8	8:7 tonus minor	231	a crack
9	9:7 ditonus latus	435	too wide

Coniectura: I do not think the seven is forbidden by nature. I think it is forbidden by us, because our singers were taught by the six and cannot find it in the throat. Give a child a canon marked in sevenths and he will sing music that his grandfather calls noise — which is what every grandfather has always called it.

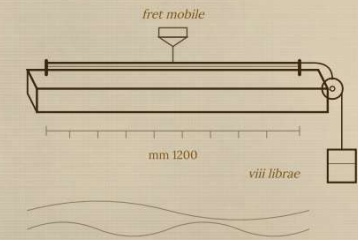


Fig. 3. The canon with its sliding fret and hanging weight of eight pounds; beneath, the first and second modes drawn as the eye seems to see them.

AD MARGINEM

In the margin

לֹא אִישׁ הַיּוֹם הַזֶּה הוֹלֵךְ בְּלִי מַחְשָׁבָה

Let no man who is not a mathematician read me. Written twice on this leaf, the first and second modes drawn as the eye seems to see them.

FRETTING BY RULE

The rule of eighteen, for a lute of 650 mm

FRET	RULE	TRUE
I	36.1	36.5
II	70.2	70.9
III	102.4	103.3
IV	132.8	133.8
V	161.5	162.6
VII	214.3	216.7
XII	320.7	325.0

Take away an eighteenth of what remains, and again, and again. The rule is quick and errs by four millimetres at the octave — enough to be heard, and the reason the good makers correct the twelfth fret by hand.

ERRATA COPISTAE

What the second hand altered, and where

LOCO	UT SCRIPTUM	CORRECTUM
col. i	9:8 maior	9:8 tonus
col. i	1140 mm	1139.1
col. ii	sexta	sexta minor
col. ii	204	203.9
tab. iii	½ pars	media pars
marg.	—	† probatum

The copyist was honest and marked his own hand with the dagger wherever he mended a number. Six mendings on this leaf; none of them touch the reasoning, only the reckoning.

CONSONANTIAE VETITAE

Ratios the ear refuses, and why

RATIO	PARTES	IUDICIUM
7:5 tritonus	583	harsh
11:8	551	uncertain
13:8	841	no name
16:9	996	tolerable
32:27	294	flat, dull
81:64	408	wide, tense

These are not consonances but they are not nothing: the ear hears in them a promise unkept. *Coniectura: a ratio offends in proportion to the number of its parts, so that offence itself is measurable.*

SENTENCE

«Harmony is born of nothing but the touching of proportional parts in one instant of time.»